Yet Another Note on "An Efficient Zero-One Formulation of the Multilevel Lot-Sizing Problem"

Michael R. Bussieck
Department of Mathematical Optimization, Braunschweig University of Technology, Pockelsstraße 14, D-38106 Braunschweig, Germany

Andreas Fink
Department of Business Administration, Information Systems and Information Management, Braunschweig University of Technology, Abt-Jerusalem-Straße 7, D-38106 Braunschweig, Germany

Marco E. Lübbecke
Department of Mathematical Optimization, Braunschweig University of Technology, Pockelsstraße 14, D-38106 Braunschweig, Germany

ABSTRACT

In "An Efficient Zero-One Formulation of the Multilevel Lot-Sizing Problem" [8] McKnew, Saydam, and Coleman claim the polynomial solvability of this particular production planning problem. Both, the proof given by McKnew *et al.* and a statement of its incorrectness by Rajagopalan [10] contain errors, and the question remains whether the approach leads to a polyomial time algorithm. We show by means of an example that solving the linear program is not sufficient.

Subject Areas: Materials Requirement Planning, Assembly Systems, Integer/Binary Program, and Computational Complexity

INTRODUCTION

In "An Efficient Zero-One Formulation of the Multilevel Lot-Sizing Problem" [8] McKnew, Saydam, and Coleman present a zero-one formulation for the solution of the following production planning problem: Given an *assembly system* of n items, instantaneous and uncapacitated production, and lead times of zero from one stage to the next, determine a production plan that minimizes the total setup and inventory holding cost over a planning horizon of T periods. The formulation presented in [8] is not flawless, however, has been corrected in [5]—for details check also the electronic source we provide at the end of this note. McKnew, Saydam, and Coleman assert that the polyhedron P defined by the given system of $O(n \cdot T^2)$ linear inequalities in $O(n \cdot T^2)$ variables is integral. Rajagopalan [10] pointed out that the proof's fundamental argument of *total unimodularity* of the corresponding coefficient matrix does not hold. Although his claim is true, since the manipulations of the coefficient matrix performed in [8] do not preserve total unimodularity, the given counterexample of Rajagopalan contains an error.

Numerical investigations of P using `PORTA` [3] helped us to identify valid counterexamples and reveal e.g. for a tiny example with $n = 2$ and $T = 4$ that about 10% of P's vertices are fractional. Still, the possibility remained that—with respect to the objective function given in [8]—at least the optimal face of P is integral. In this case, polynomial time methods for solving linear programs [12] would provide us with a polynomial time algorithm for solving the considered multilevel lot-sizing problem. KUIK and SALOMON [7] give a single-item example that does not only eliminate this possibility in general, even worse, given their particular objective function c^T, there is a strictly positive gap between

$$z_1 := \min\{c^T x \mid x \in P, x \text{ binary}\} \qquad \text{and} \qquad z_2 := \min\{c^T x \mid x \in P\}.$$

In other words, the optimal face of the polyhedron induced by their counterexample contains *no* integral vertex at all and we have to resort to e.g. branch and bound techniques to find a binary optimal solution. However, in order to obtain this result they need to introduce *negative* inventory holding cost. It still remains open whether an efficient solution of *practically relevant* instances (non-negative setup and inventory holding cost) is possible with the approach by MCKNEW *et al.* We close this gap and give a negative answer to the question by means of the following example.

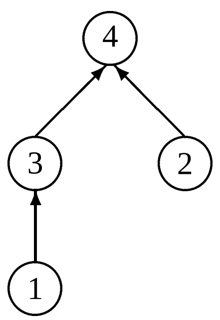

Fig. 1. Assembly system

Consider the assembly system of four items depicted in Figure 1, a planning horizon of eight periods, a constant demand for item 4 of 300 per period, and a particular setting of setup and inventory holding cost.

	item 1	item 2	item 3	item 4
setup cost	1	1	2,000	1,500
inventory holding cost	1	1	2	4

The fractional optimal solution of the formulation under consideration provides an objective function value $z_2 = 18{,}756.5$ whereas the optimal *binary* solution takes on the value $z_1 = 18{,}808$.

The corresponding model based on the formulation given in [8] encoded—in it's corrected form—in the modeling languages GAMS [2] and AMPL [6], respectively, as well as the `PORTA` files are publicly available at URL `ftp://ftp.math.tu-bs.de/pub/lot/counter.tar.gz`.

CONCLUSION

ARKIN *et al.* [1] state that the multi-level lotsizing problem in assembly systems is polynomially solvable. They refer to a technical report by RAJAGOPALAN and CORNUÉJOLS [11], which, according to CORNUÉJOLS [4] contains an error. Neither a comprehensive literature review nor an inquiry to relevant researchers in the field [9] did reveal a polynomial time algorithm or a proof of NP-completeness of the problem, respectively. In this note we showed that the approach by MCKNEW, SAYDAM, and COLEMAN is not polynomial, either. Thus, the computational complexity status of the problem remains unsettled.

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Michael R. Bussieck is a research assistant in the Department of Mathematical Optimization at the Braunschweig University of Technology, Germany, from where he received his M.S. in mathematics. His research interests are in large scale integer programming problems, modeling systems, and real-world applications of combinatorial optimization. His research has been published in *Computational Geometry: Theory and Applications*, *European Journal of Operational Research*, *Mathematical Methods of Operations Research*, *Mathematical Programming*, and *Operations Research Letters*.

Andreas Fink is a research and teaching assistant in the Department of Business Administration, Information Systems and Information Management at the Braunschweig University of Technology, Germany. He holds a diploma degree in Computer Science and Business Administration from the Darmstadt University of Technology, Germany. He published in *Computers & Operations Research* and *OR Spektrum*. His research interests include modern heuristic search methods, production planning, and software development.

Marco E. Lübbecke is a research assistant in the Department of Mathematical Optimization at the Braunschweig University of Technology, Germany, from where he received his M.S. in mathematics. His research interests are in combinatorial optimization and large scale integer programming problems, especially arising from transportation and logistics applications. He published in *Computational Geometry: Theory and Applications*. Currently, he is mainly involved in locomotive scheduling at industrial freight railroad companies.